
Research of the Design and Practice of Mathematical Modeling
Learning Scaffold in Senior High School

20204507046

学位论文数据真实性和原创性承诺

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指导教师关于学生论文真实性审核的声明

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关键词:

Research of the Design and Practice of Mathematical Modeling Learning Scaffold in Senior High School

Abstract

As a tool to solve practical problems, mathematical modeling is an effective way to develop and maintain student's enthusiasm and curiosity about mathematics, which has great educational potential and value. However, through sorting out 26 investigations on mathematical modeling, it could conclude that there are three common problems in the teaching process of mathematical modeling in senior high schools in China: (1) Lack of school's attention; (2) Lack of relevant teachers; (3) Lack of student's experience, and most of them have not even learned about it. Therefore, via reading relevant literature, this paper aimed to design a teaching method scaffold teaching method, which is suitable for beginners in the learning of mathematical modeling, in order to promote the capacity of senior high school students for this subject.

The American High School Mathematical Modeling Contest (HiMCM), a non-profit organization in the United States, is an international mathematics competition sponsored by COMAP. In this paper, combined the award-winning papers of HiMCM students, the preliminary form of mathematical modeling scaffold was designed through text analysis. Then, modified the scaffold's frame to obtain the mathematical modeling scaffold via interviewing four students who won the first prize of HiMCM in the practice school in 2020. Finally, practice the use of scaffold in modeling community in school.

After carrying out the experiment of the whole process in school, and basing on the data analysis of students' mathematical modeling ability and comprehensive level, it can conclude that: (1) Comparing the control group, the students in experiment group

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showed better process in the ability of simplification, mathematization and testing; (2)There was no significant difference between the two groups in mathematical modeling.It shows that the scaffold has little effect on students' mathematical modeling ability in the short term; (3)In the pre-test, there was no significant statistical difference between students' mathematical modeling ability and mathematical achievement,However,After studying in the mathematical modeling community, students' mathematical achievement is positively correlated with their mathematical modeling ability; (4)Scaffolds are helpful for students with common mathematical modeling ability, but not for students with excellent mathematical modeling ability. Based on the above conclusions, there is no denying that scaffolds can help beginners in mathematical modeling to quickly improve their abilities of simplification, mathematization and testing, and the effect is more obvious for students with lower mathematical modeling ability.

Keywords:Mathematical modeling; Scaffold teaching methods;senior; high school student

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Supervised by:

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[8] Maaß K Mathematisches Modellieren im Unterricht[J] Ergebnisse einer empirischen Studie Hildesheim Franzbecker Verlag 2004

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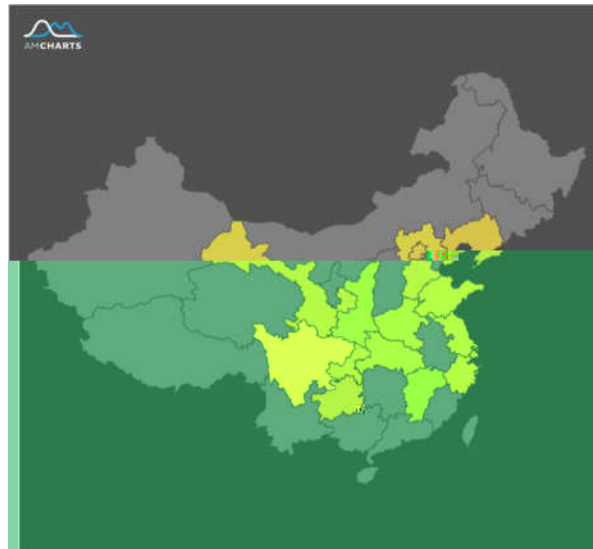
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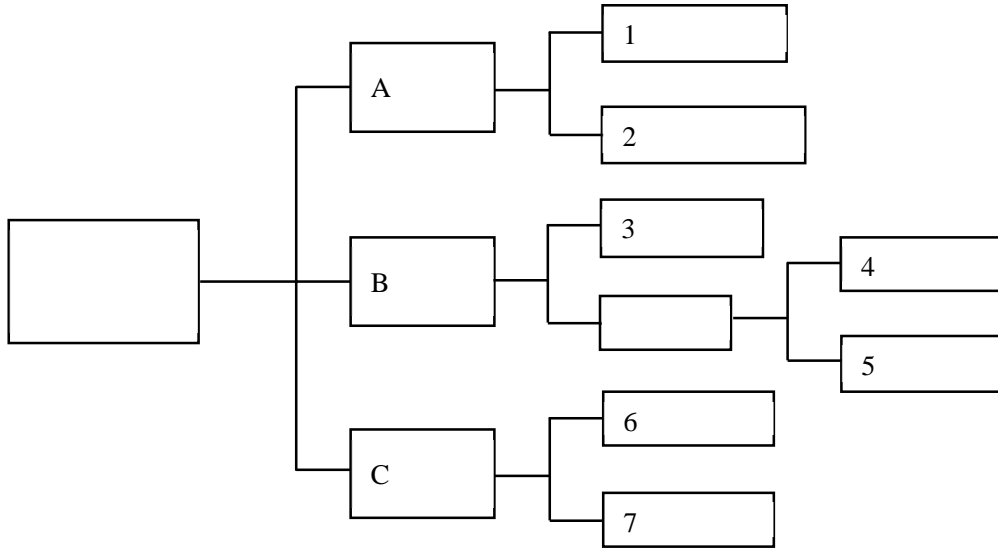
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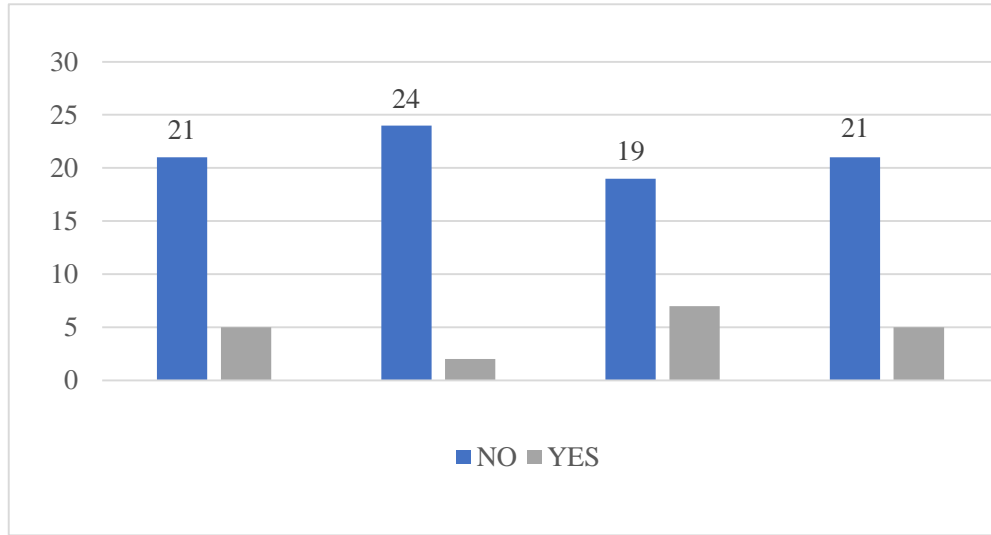
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[41] Wood D Bruner J & Ross G The role of tutoring in problem solving[J] Journal of Child Psychiatry
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[51] Zöttl L Ufer S. & Reiss K. Modelling with heuristic worked examples in the KOMMA learning environment[J] Journal f r Mathematikdidaktik 2010 31(1):143-165

[52] Catharina Beckschulte Mathematical Modelling Education and Sense-making[M]. Gloria Ann Stillman Gabriele Kaiser Christine Erna Lampen (Eds.) 2020:129-139



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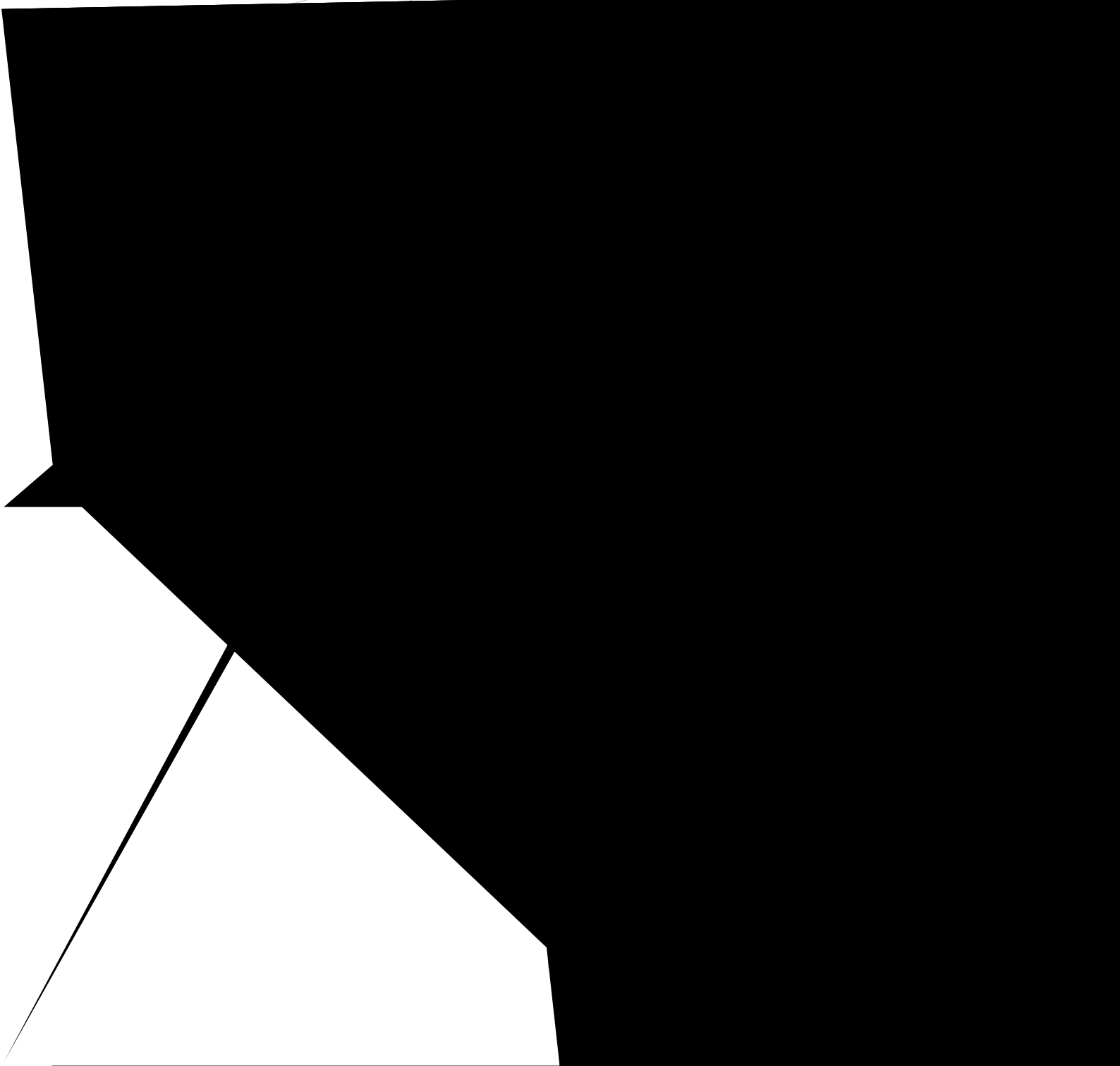
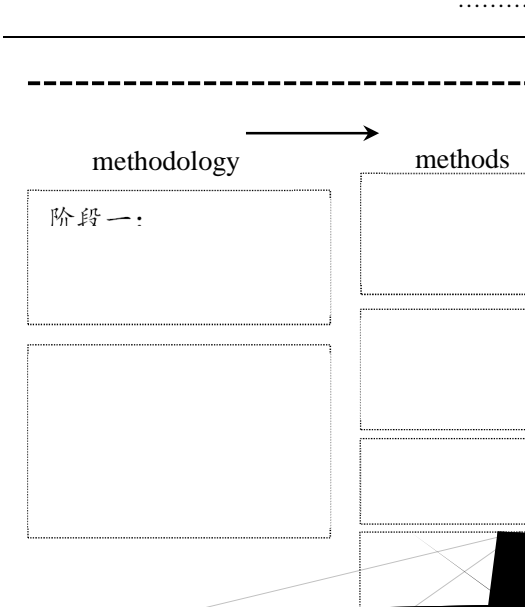
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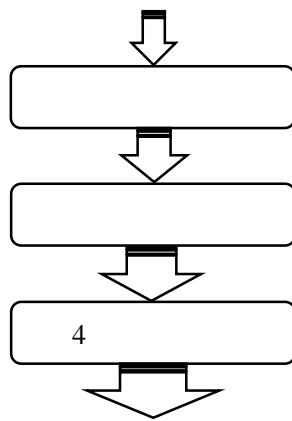
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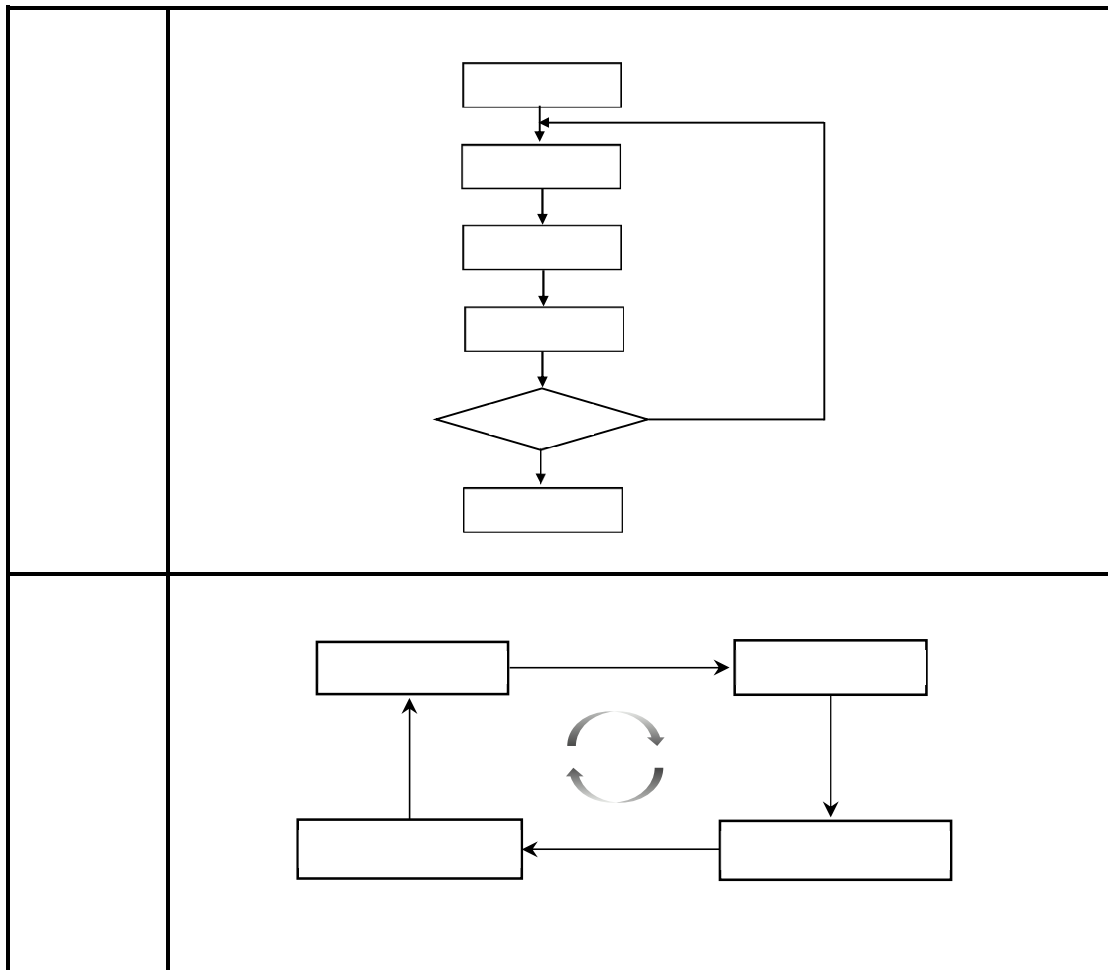
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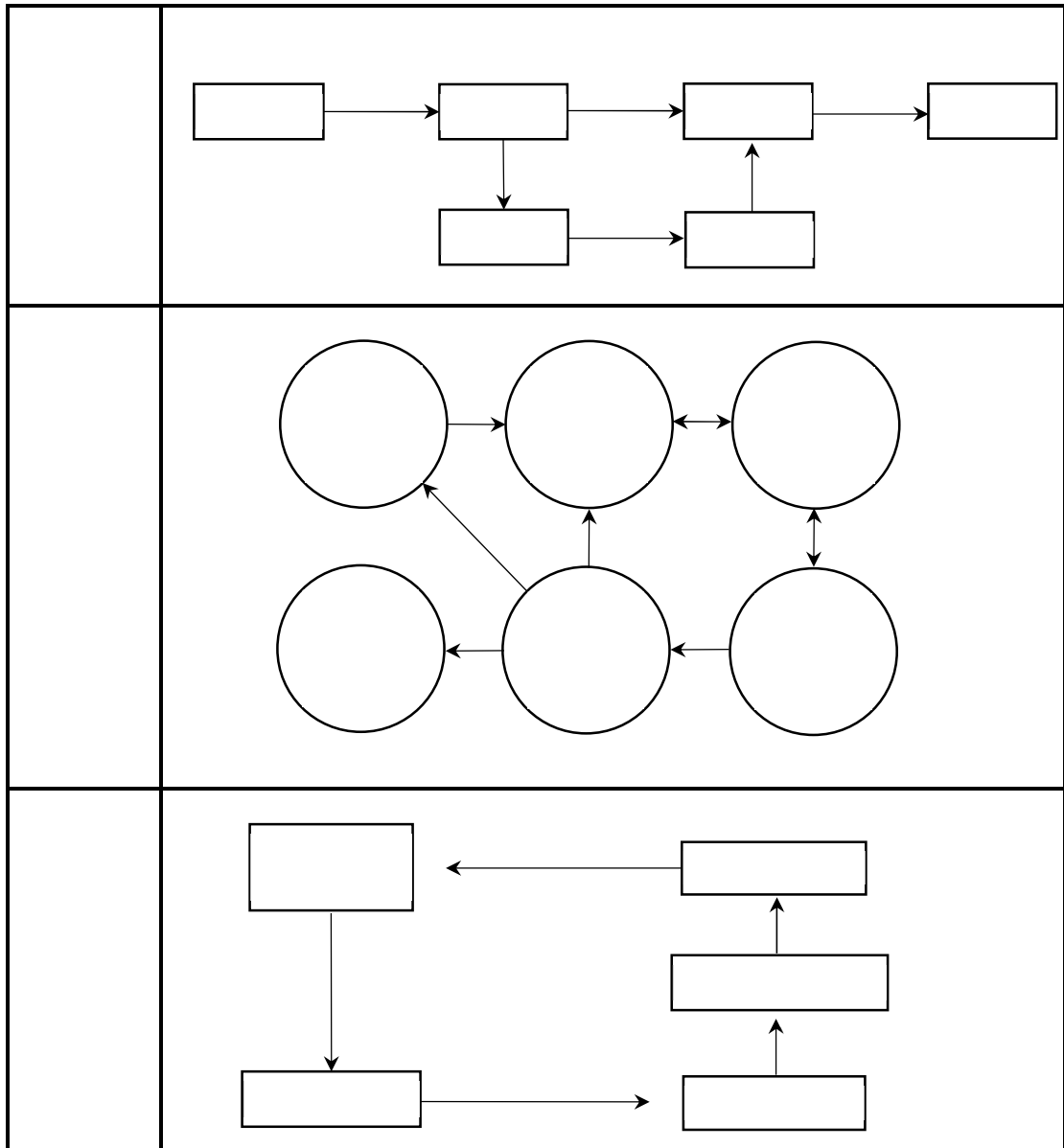
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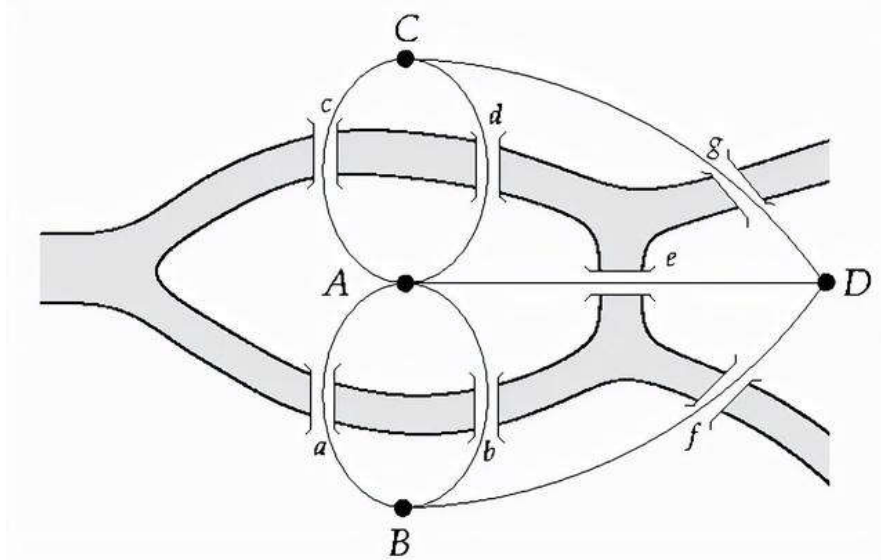
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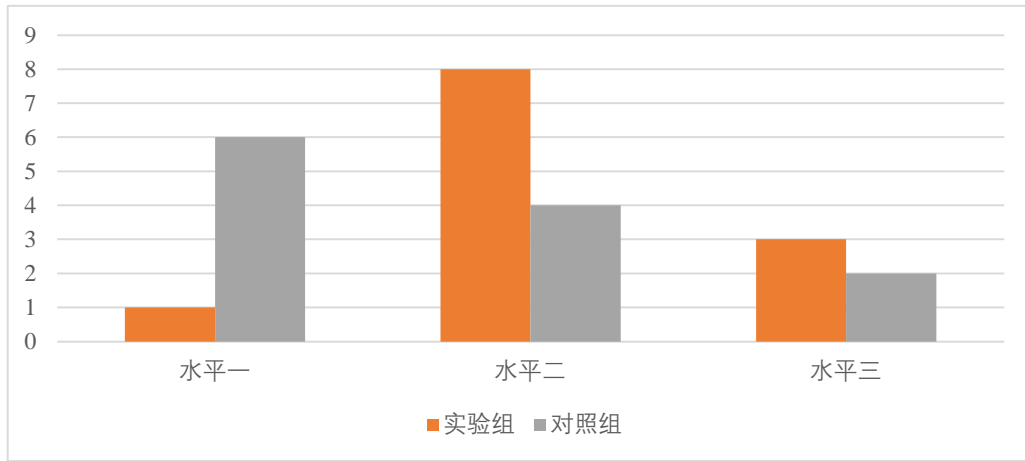
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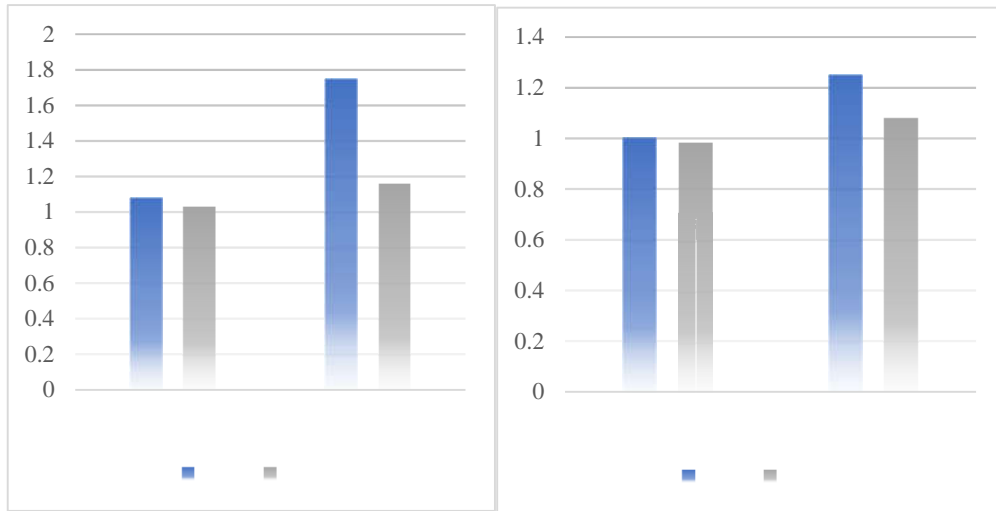
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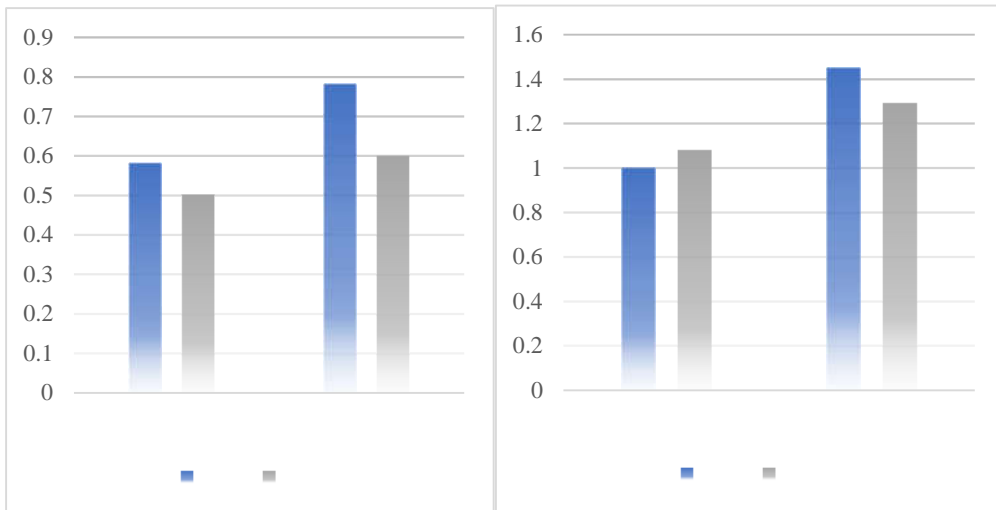
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-
- [31] Blum W Quality teaching of mathematical modelling: What do we know
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 M S Biembengut (Eds) Mathematical modelling in edu- cation research
 and practice 2015 73-96 Cham: Springer
- [32] [D] 2020
- [33] [D] 2019
- [34] [D] 2020
- [35] [D] 2019
- [36] [D] 2018
- [37] [J] 2008 47 11

- [47] Schukajlow S Kolter J &Blum W Scaffolding mathematical modelling with a solution plan ZDM Mathematics Education 2015 47(7) 1241-1254
- [48] Smit J van Eerde H A &Bakker A A conceptualisation of whole-class scaffolding[J] British Educational Research Journal 2013 39(5):817-834
- [49] Blum W Can modelling be taught and learnt? Some answers from empirical research[M] In G Kaiser W Blum R Borromeo Ferri & G Stillman (Eds) Trends in teaching and learning of mathematical modelling 2011:15-30 Dordrecht: Springer
- [50] Blum W Can modelling be taught and learnt? Some answers from empirical research[M] In G Kaiser W Blum R Borromeo Ferri &G Stillman(Eds) Trends in teaching and learning of mathematical modelling 2011:24 Dordrecht: Springer
- [51] Zöttl L Ufer S. & Reiss K. Modelling with heuristic worked examples in the KOMMA learning environment[J] Journal für Mathematikdidaktik 2010 31(1):143-165
- [52] Catharina Beckschulte Mathematical Modelling Education and Sense-making[M]. Gloria Ann Stillman Gabriele Kaiser Christine Erna Lampen (Eds.) 2020:129-139
- [53] [M] 1987:111-113
- [54] [J] 2020 29 5 10:52
- [55] [M] 2015:163
- [56] [M] 2015:166
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- A t
- B t
- C t
- D
- E

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- A $1-e^{-t}$
- B $(1-t)^2$
- C t
- D $t-t^2$
- E $1/1-e^{-t}$

6

- | | | |
|-------|-------|-------|
| n_1 | m_2 | n_2 |
| t | p | |

?

- A $m_1(p + n_1t) = m_2(p + n_2t)$
- B $m_1(p + n_1t) < m_2(p + n_2t)$
- C $m_2(p + n_2t) \leq m_1(p + n_1t)$
- D $m_2(p + n_2t) < m_1(p + n_1t)$
- E $m_1(p + n_1t) \leq m_2(p + n_2t)$

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邮件 1: 12月7日上午 10:00	邮件 2: 12月7日上午 10:30
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9	: 1 ? ? 2 ? 3 ?
邮件 3: 12月7日下午 13:00	邮件 4: 12月7日下午 5:00
1 5 , 15 , 5 25	
2 60 60 60 60 60	
3 60	

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